Week 13: Math Review
Logs & Exponents

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Why are we reviewing this... 😬

Part of learning about data analysis and statistics is understanding the "how" and the "why", to peek under the hood and see how things work

This includes (partially) deriving some equations we use mathematically

And whenever we do that, we often have to work some ✨math magic✨

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https://bookdown.org/ndphillips/YaRrr/the-structure-of-a-custom-function.html

THROWBACK!

Solve for x:

$4(x - 2) = 2x + 6$

Distribute the 4 (multiply)

$4x - 8 = 2x + 6$

Add 8 to both sides

$4x = 2x + 14$

Subtract 2x from each side

$2x = 14$

Divide both sides by 2

$x = 7$

In order to manipulate equations (e.g., to isolate x), we often need to "undo" things -- do the opposite or the inverse!

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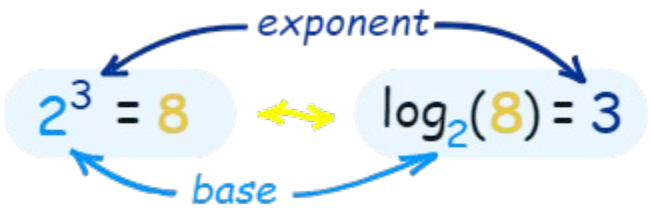
Another example:

Solve for x:

$2^x = 8$ We need logs!



Exponents say: "how many times do I multiply the base by itself?"
E.g., $2^3 = 2 \times 2 \times 2 = 8$

Logs say: "what exponent produced this?"
E.g., $2^x = 8$
$x = \log(8) = 3$

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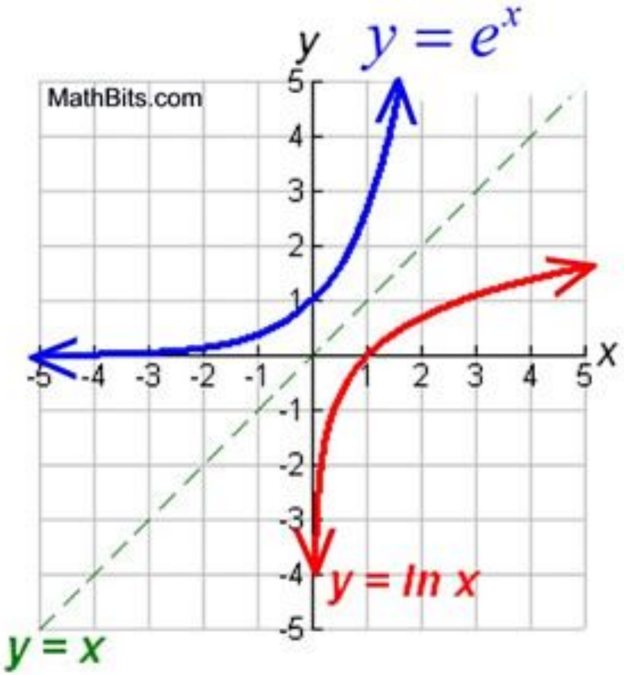
Euler’s e and natural logs

In statistics we use *e* as our base for exponents, the ‘natural exponent’

e is a constant, and one of the most important numbers in math! It has a ton of fun and important properties

e = 2.7182818284590

$\log_e x$ = lnx (the natural log)



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Properties of logs and exponents

$\ln_e x = x$

$e^{\ln x} = x$

Operation	Laws of exponents	Laws of logs
Multiplication	$x^m \cdot x^n = x^{m+n}$	$\log(a \cdot b) = \log(a) + \log(b)$ ★
Division	$\frac{x^m}{x^n} = x^{m-n}$	$\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$
Exponentiation	$(x^m)^n = x^{mn}$	$\log(a^n) = n \cdot \log(a)$ One of the most useful properties of logs
Zero property	$x^0 = 1$	$\log(1) = 0$
Inverse	$x^{-1} = \frac{1}{x}$	$\log(x^{-1}) = \log\left(\frac{1}{x}\right) = -\log(x)$

It is a lot easier to deal with and calculate sums than it is to work with multiplication!

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Practice

Let's use a property to make our lives easier!

Imagine needing to calculate: 1 * 2 * 3 * ... * 10000

That number is going to get big really quickly! One strategy would be to take the log of equation, and then use the multiplication property to simplify:

= log(1 * 2 * 3 * ... * 10000)

= log(1) + log(2) + log(3) + ... + log(10000)

$$= \sum_{i = 1}^{10000} \log(i)$$

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Normal Distribution Formula

$\mu =$ mean of x

$\sigma =$ standard deviation of x

$\pi \approx 3.14159 \ldots$

$e \approx 2.71828 \ldots$

$$f(x) = \boxed{\frac{1}{\sigma\sqrt{2\pi}}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

Constant!

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